# Spectral Redundancy in a 6-Ink Ink-Jet Printer

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## Abstract

It is demonstrated for the multi-ink printers investigated that there are many spectral reflectances that the same printer can approximately produce through a large variety of different ink combinations. This *spectral redundancy* was evaluated for a pair of 6-ink ink-jet CMYKGO printers. For each printer, more than twenty thousand samples of ink combinations were printed and measured with a spectrophotometer. Fifteen thousand of the samples populated 6-dimensional lookup tables used to convert fractional area coverage to reflectance spectra. Through use of the lookup tables, density maps were built illustrating the 6-dimensional distribution of redundancy throughout colorant space. The setting of tolerances and the choice of tolerance metrics is application specific. If spectral RMS difference were the chosen metric, at a tolerance of 0.01 RMS, none of the inks in our CMYKGO systems were fully replaceable by combinations of the other inks. However, when the tolerance is doubled to 0.02 RMS, the degrees of freedom for matching spectra in the systems fall to five because the five chromatic inks cover the entire spectral gamut without the need of the black ink. Systematic relationships among the inks are reported detailing the likelihood that combinations of printer digital counts may be replaced by largely different ones while preserving spectral reflectance to within an RMS tolerance.

## Introduction

For multi-ink printers, it is found that approximate many-to-one relationships exist between combinations of inks printed on paper and the measured spectral reflectances of the prints. This we call *spectral redundancy*. Figures 1, 2, 3 and 11 show examples of pairs of different ink combinations with similar printed reflectance spectra. Since there are many sources of uncertainly within any printing and measurement system, two reflectances can only be said to match when they do so within a tolerance. Tolerances may be set based on an error analysis of a system and may also be based on an application's specific requirements.

Color reproduction chains are already well known for their redundant aspects. For example, colorimetry can often be maintained when an original combination of printing inks is replaced by an appropriate alternative combination of printing inks. For four ink printers, methods built around colorimetric redundancy include GCR and UCR where black ink is swapped in for some amounts of chromatic inks. Results of the current investigation illuminate a phenomenon with many analogies to the observations underlying successful gray replacement algorithms, but instead of holding appearance constant, it is shown that in many cases the more fundamental property of spectral reflectance may be approximately maintained while drastically modifying ink levels.

For these investigations, spectral RMS difference was the chosen method for determining a match between spectra. For many applications, other metrics may be far more appropriate. The investigation of metrics for spectral reproduction applications is an ongoing area of research[1].

Determining the presence of wide-spread spectral redundancy within a printing system is an important discovery in itself. For those developing means of efficient image processing for spectral color reproduction[2], it raises important cautions when considering the use of traditional color management building-blocks such as multi-dimensional lookup tables. Further, it opens the gates to spectral reproduction systems that manage imaging characteristics beyond spectra such as maximizing color constancy, minimizing total ink coverage, or controlling the use of individual inks. This may also point toward the development of criteria for design of inks in spectral reproduction systems.

## Spectral Redundancy in a CMYKGO Printer

An Epson Stylus Photo 1200 6-ink ink-jet printer was retrofitted to print with four standard process inks plus an orange and a green ink. The characterization process was discussed in Reference 3. During the investigation documented there, it was found that a 6-dimensional characterization LUT with 5 nodes per dimension (5x5x5x5x5x5) was accurate for converting from fractional area coverage to estimated spectra of a printed patch.

When inverting through the 6-dimensional characterization LUT there were some surprises. Although unpublished until now, it was found during the previous investigation that by slightly modifying the inversion parameters, it was possible to produce a variety of different ink specifications that well matched the same goal reflectance. Printing the chosen ink digital counts showed that the reflectances for the different ink specifications were well predicted and did indeed produce nearly identical reflectances. See Figures 1, 2 and 3. Table I explains the ink combinations used to make the examples described in the figures. Table II shows the RMS difference between the measurements from the sample pairs.

**Table I. Ink Combinations for Figures 1–3**

| Fig. | Sample | Fractional Ink Coverages | | | | | |
|---|---|---|---|---|---|---|---|
| | | C | M | Y | K | G | O |
| 1 | 1 | 0.00 | 0.50 | 0.75 | 0.50 | 0.00 | 0.50 |
| | 2 | 0.37 | 0.56 | 0.81 | 0.00 | 0.14 | 0.63 |
| 2 | 3 | 0.25 | 0.00 | 0.75 | 0.25 | 0.75 | 0.00 |
| | 4 | 0.22 | 0.05 | 0.76 | 0.17 | 0.78 | 0.02 |
| 3 | 5 | 0.25 | 1.00 | 0.50 | 1.00 | 0.00 | 0.00 |
| | 6 | 0.67 | 1.00 | 0.75 | 0.82 | 0.64 | 0.79 |

**Table II. RMS Differences for Figures 1-3**

| Fig. | Samples | RMS Difference |
|---|---|---|
| 1 | 1 & 2 | 0.011 |
| 2 | 3 & 4 | 0.006 |
| 3 | 5 & 6 | 0.010 |

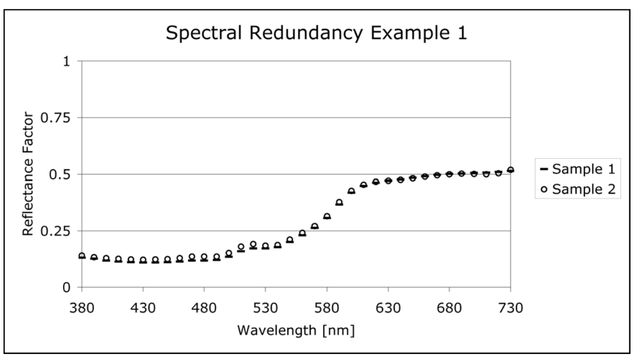


*Figure 1. Measured spectral reflectances from samples 1 and 2.*

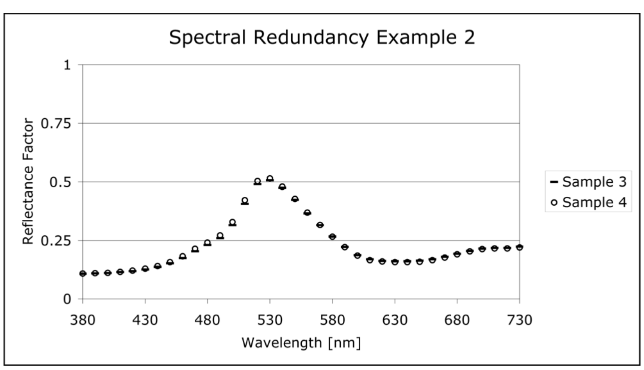


*Figure 2. Measured spectral reflectances from samples 3 and 4.*

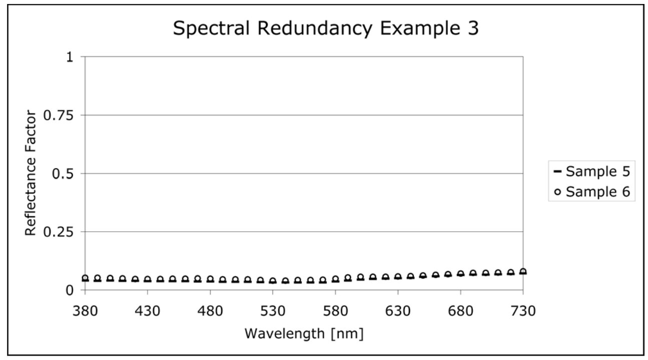


*Figure 3. Measured spectral reflectances from samples 5 and .6.*

## Spectral Independence of the Inks

When the above observations came to light, an immediate question arose as to whether all 6 inks used in the system were linearly independent of all the others. Given the presence of a green and an orange, it was possible that the reflectance characteristics of one or both could be generated from some combination of the other inks.

To study this question, a new version of the routine for inverting through the characterization LUT was implemented to optimize the best ink combination for a requested spectrum while holding a specified ink to 0. For example, to see if the reflectance properties of the green ink were linearly related to a combination of the five other inks, the spectral reflectances of the green ramp were inverted by the routine while disallowing any participation of the green ink itself. The green ramp consisted of patches formed by printing only green from 12.5% area coverage to 100% area coverage, stepped in increments of 12.5%.

Results for the ramps are found in Figures 4 through 9. Table III reports the RMS differences between the ramp spectral reflectances and the estimated reflectances from the inverted ink combinations.

Analysis of the RMS data in Table III shows that green and orange cannot be completely replaced by any combination of the other inks. For both, there is more than 0.010 RMS difference as the other inks attempt to emulate the spectra measured at 1.0 fractional area coverage. Table III shows the story to be similar for the other chromatic inks as well. It is easy to see in Figures 6 through 8 that the matches are overall quite poor.

Pure black, on the other hand, is shown to be not as hard to emulate. A maximum RMS difference of 0.019 for matching the black ramp is reported in Table III. This maximum error falls at 0.50 fractional area coverage. Figure 9 shows systematic differences between the combined matching inks and the black reflectances, especially in the low- and mid-wavelengths and in the mid-area coverages. The differences become well within measurement error as area coverage becomes very small or very large.

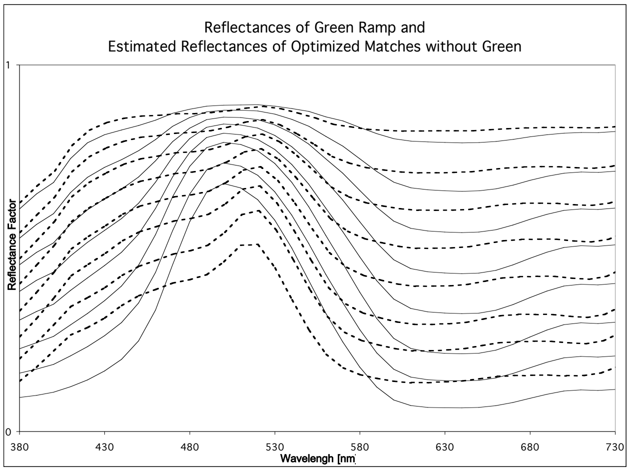


*Figure 4. Checking the linear independence of the Green ink. Solid lines: Green ramp; broken lines: estimated matches.*

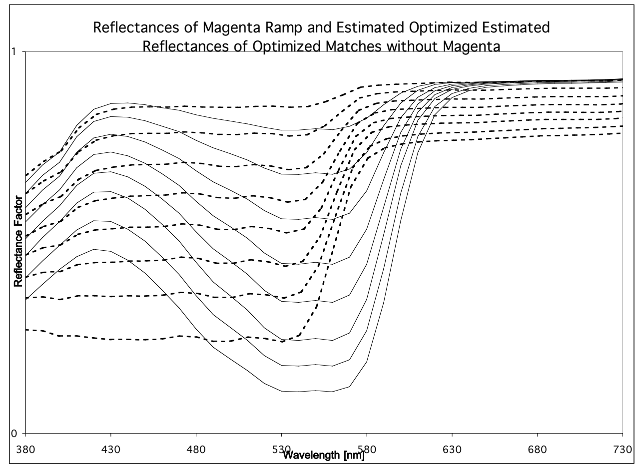


*Figure 7. Checking the linear independence of the Magenta ink. Solid lines: Magenta ramp; broken lines: estimated matches.*

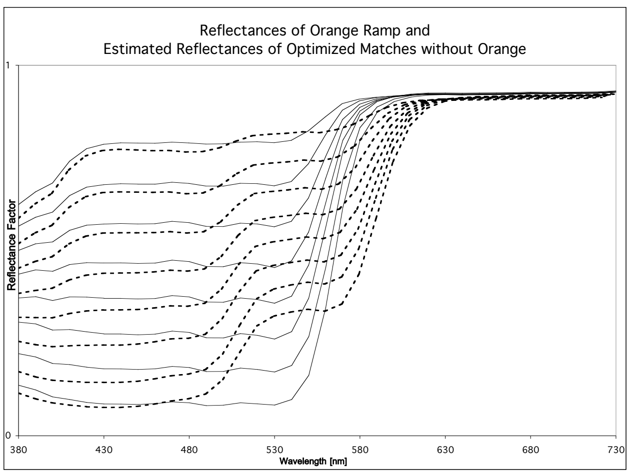


*Figure 5. Checking the linear independence of the Orange ink. Solid lines: Orange ramp; broken lines: estimated matches.*

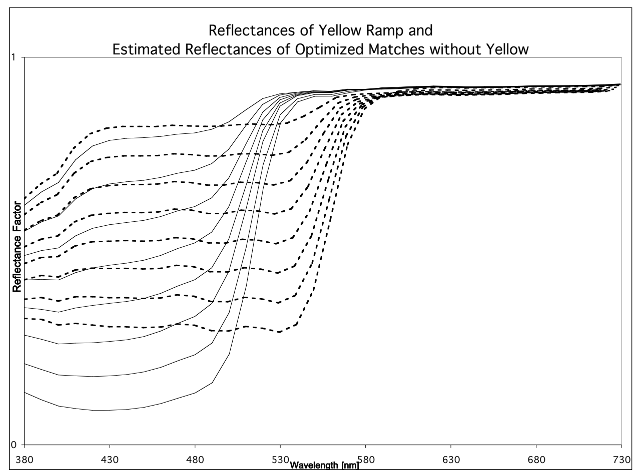


*Figure 8. Checking the linear independence of the Yellow ink. Solid lines: Yellow ramp; broken lines: estimated matches.*

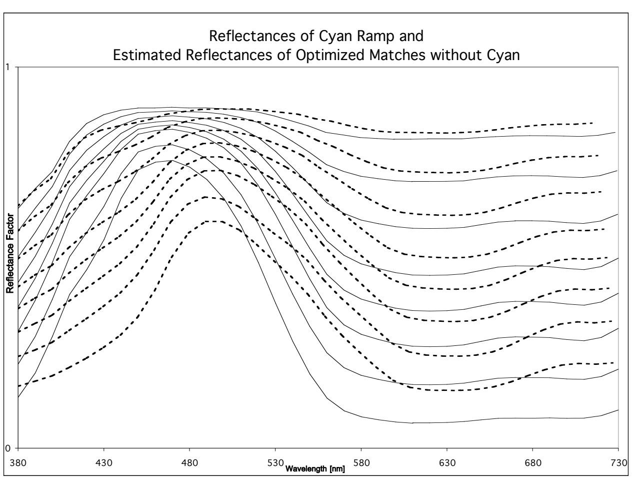


*Figure 6. Checking the linear independence of the Cyan ink. Solid lines: Cyan ramp; broken lines: estimated matches.*

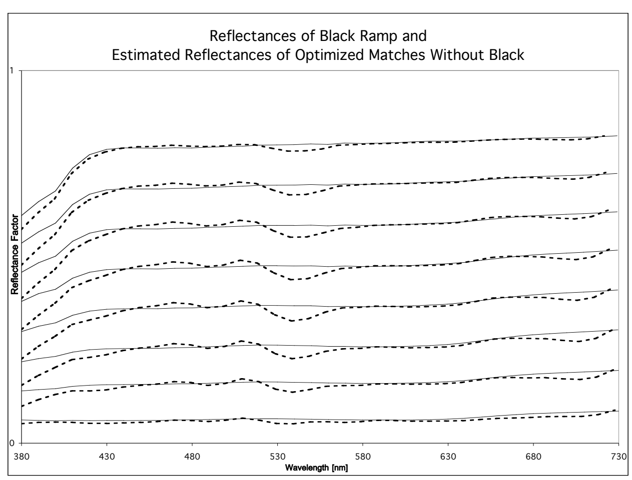


*Figure 9. Checking the linear independence of the Black ink. Solid lines: Black ramp; broken lines: estimated matches.*

0.02 RMS difference is twice our within-sheet repeatability error. If that were chosen as a spectral matching tolerance, black might be considered as being linearly dependent to the other inks. In the next section analysis goes beyond the pure color ramps and determines the redundancy of the inks in the presence of other inks.

**Table III. RMS Differences for Figures 4-9**

| Ramp Level | Ramp | | | | | |
|---|---|---|---|---|---|---|
| | G | O | C | M | Y | K |
| **0.125** | 0.018 | 0.019 | 0.024 | 0.032 | 0.030 | 0.009 |
| **0.250** | 0.037 | 0.038 | 0.048 | 0.065 | 0.061 | 0.015 |
| **0.375** | 0.052 | 0.055 | 0.074 | 0.096 | 0.088 | 0.018 |
| **0.500** | 0.067 | 0.073 | 0.101 | 0.128 | 0.116 | 0.019 |
| **0.625** | 0.081 | 0.088 | 0.121 | 0.153 | 0.142 | 0.018 |
| **0.750** | 0.097 | 0.104 | 0.142 | 0.179 | 0.169 | 0.015 |
| **0.875** | 0.099 | 0.111 | 0.153 | 0.191 | 0.190 | 0.010 |
| **1.000** | 0.106 | 0.122 | 0.172 | 0.205 | 0.212 | 0.004 |

## Distribution of Spectral Redundancy in Colorant Space

A systematic approach to mapping out the density of spectral redundancy was undertaken. A second 6-ink printer, an Epson Stylus Pro 5500 with a different set of CMYKGO inks and on different paper was characterized in a similar manner[3] as done for the experiments reported above. A set of ink combinations spanning colorant space, refered to as the *midpoint set*, was printed during characterization. For this study the set was exploited to determine the density distribution of spectral redundancy.

The midpoint set consists of the midpoints of each one of the 4096 6-dimensionsal hypercubes that make up the characterization LUT. It was originally designed to be used in evaluating the robustness of the characterization LUT. It included all combinations of CMYKGO inks with the following fractional area coverages:

0.125, 0.375, 0.600, 0.875

A consequence of this sampling is that all inks are present with at least 0.125 fractional area coverage in every printed patch of the midpoint set.

To study redundancy the measured reflectance for each midpoint was fully probed. For each measured spectrum, the characterization LUT was inverted 1536 times. Each time the LUT was inverted, one of the six ink digital counts was held to a value between 0 and 255 while the other five inks were allowed to vary to any level. Minimum RMS differences between the spectrum associated with the optimized digital counts and the measured spectrum for that midpoint were recorded. In this way every digital count for every ink was tested for every midpoint spectrum.

For example, Figure 10 shows a redundancy profile derived through the process. Here the original digital counts for the CMYKGO midpoint patch were respectively 7, 6, 39, 3, 179 and 5. The figure shows the minimum RMS difference when holding individual inks to particular x-axis values. Within Figure 10, there are six difference plots, each associated with one of the six inks. Notice plot minima are found at or near the point where the x-axis is equal to each original digital count. This is due to a small amount of interpolation error in the characterization LUT and the fact that the inversion routine will quit when RMS difference is sufficiently low.

As each plot moves away from an ink's original digital counts, RMS difference tends to rise due to the system's increased difficulty in matching the original midpoint spectral reflectances with the fixed ink level of the controlled ink.

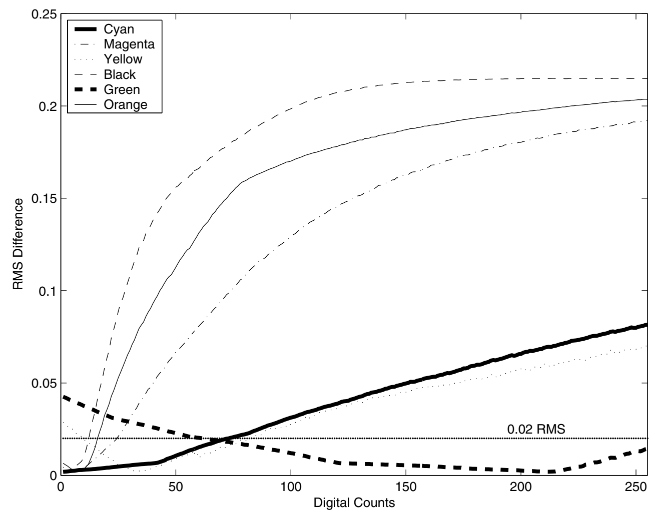


*Figure 10. Error profile for midpoint reflectance from original digital counts C= 7, M=6, Y=39, K=3, G=179 and O= 5.*

Figure 11 shows two reflectance spectra. The first is a measurement of a printed patch of the original digital counts of Figure 10. The second is a measurement of a printed patch of the digital counts associated with where the cyan curve crosses 0.02 RMS in Figure 10. Those digital counts are CMYKGO 71, 0, 62, 0, 70 and 10, respectively. Although the resultant spectra are very close, the Euclidean distance in digit space is 112.16 digital counts. After printing and averaging the measurement of two samples, the RMS difference between the two is close to the prediction at 0.018.

Figure 12 is similar to Figure 10 except it summarizes results for all 4096 midpoints. It shows for each ink the maximum differences from the entire set at each digital count. Of great interest are the values at digital count of 0 because that is where there is no participation at all from the particular ink. Significantly, Figure 12 shows that for the entire midpoints set, black can be held to 0 without introducing RMS spectral difference above 0.02. Thus, for an RMS tolerance of 0.02, black is found to be completely spectrally redundant to the other five inks for the midpoint set of reflectance spectra.

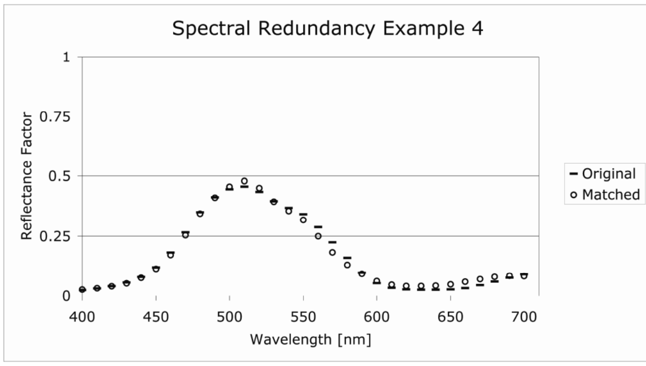


*Figure 11. Measured spectral reflectances for Figure 10's original CMYKGO and its match where cyan crosses .02 RMS.*

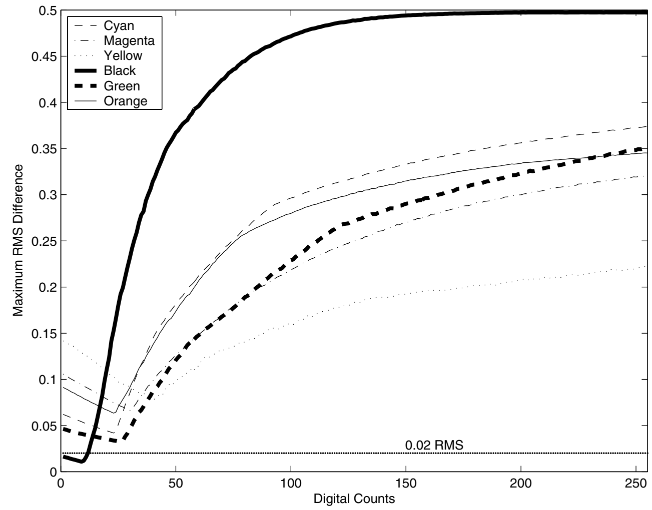


*Figure 12. Maximum error differences for entire midpoint set.*

For an RMS tolerance of 0.02, the 4096 midpoint redundancy analyses yielded the redundancy density maps of Figures 14 through 19. These are complicated graphs to read. Figure 13 explains how to read them. The gray scale values indicate the range of digital values under 0.02 RMS. For example, Cyan in Figure 10 has a range of 71 under 0.02 RMS.

Table IV summarizes conclusions based on the density maps.

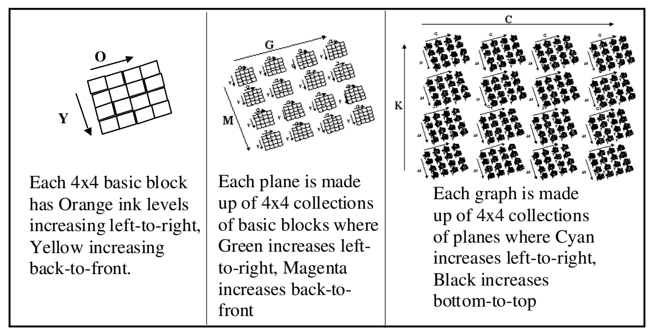


*Figure 13. Description of how to interpret the positions in Figures 14 through 19. The ink levels are with respect to the original midpoint CMYKGO. The grayscale level indicates the digital range for error less than 0.02 RMS.*

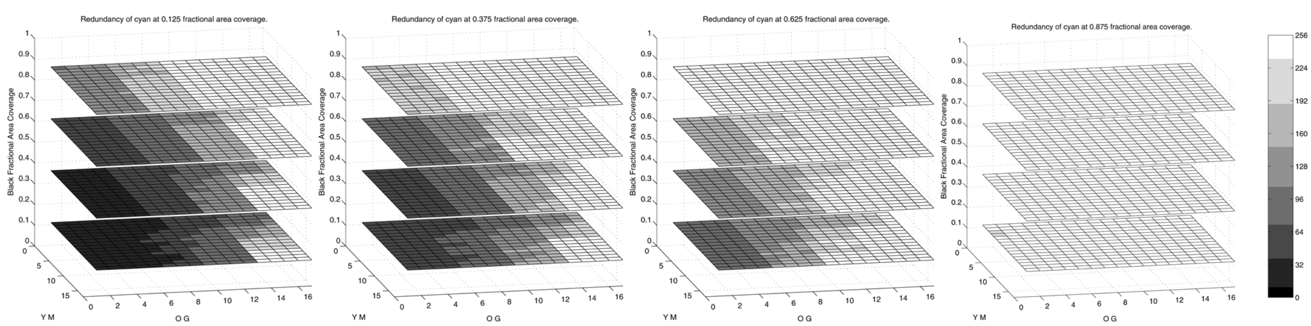


*Figure 14. Cyan redundancy density map. Gray level indicates the range of Cyan digital counts that can match original midpoints spectra at an RMS difference of 0.02 or less. See Figure 13 for positional interpretation information.*

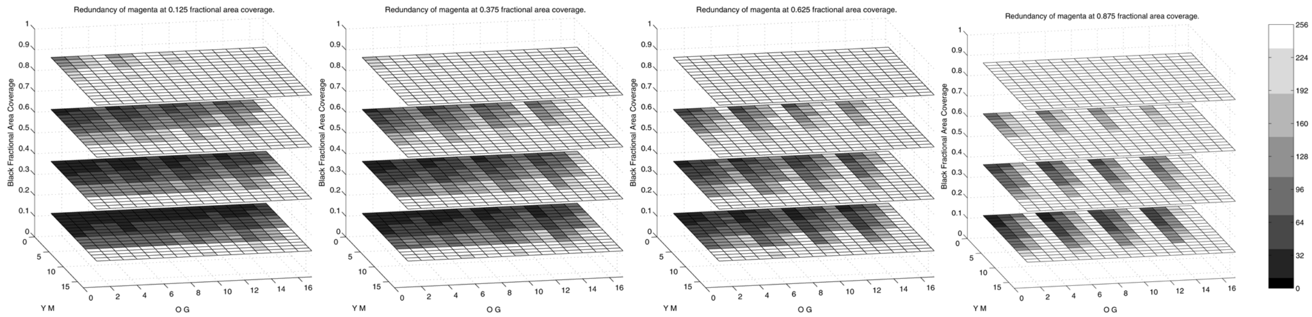


*Figure 15. Magenta redundancy density map. Gray level indicates the range of Magenta digital counts that can match original midpoints spectra at an RMS difference of 0.02 or less. See Figure 13 for positional interpretation information.*

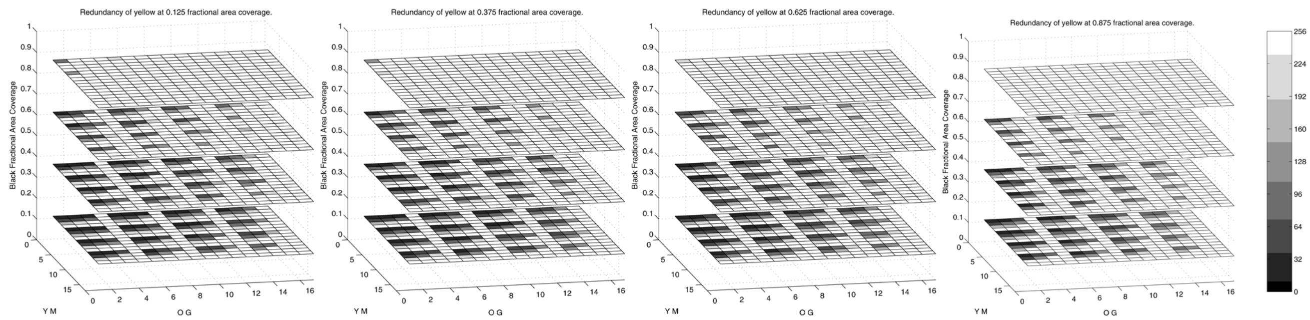


*Figure 16. Yellow redundancy density map. Gray level indicates the range of Yellow digital counts that can match original midpoints spectra at an RMS difference of 0.02 or less. See Figure 13 for positional interpretation information.*

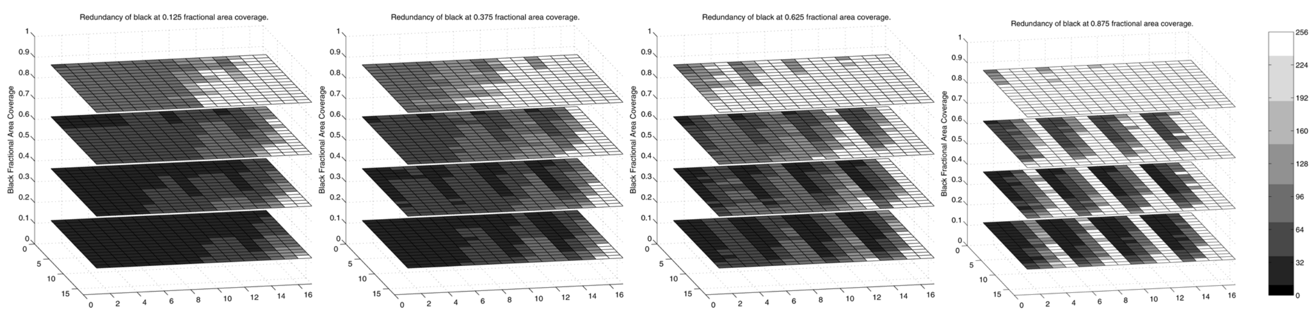


*Figure 17. Black redundancy density map. Gray level indicates the range of Black digital counts that can match original midpoints spectra at an RMS difference of 0.02 or less. See Figure 13 for positional interpretation information.*

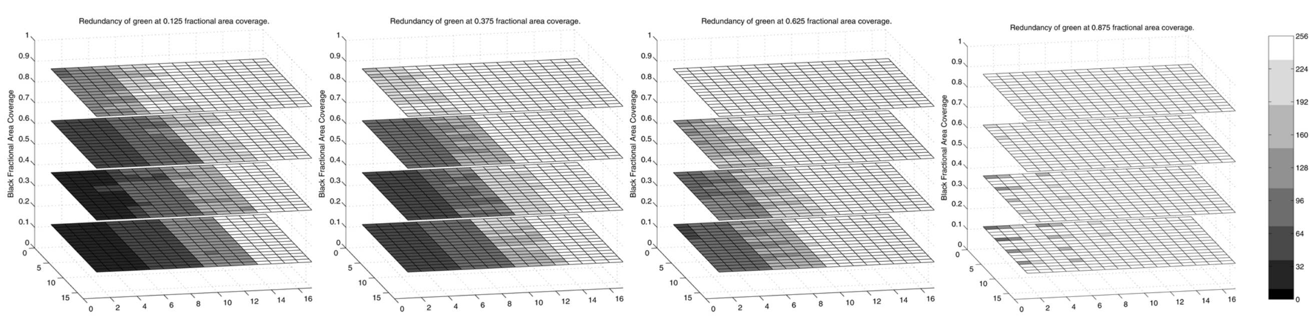


*Figure 18. Green redundancy density map. Gray level indicates the range of Green digital counts that can match original midpoints spectra at an RMS difference of 0.02 or less. See Figure 13 for positional interpretation information.*

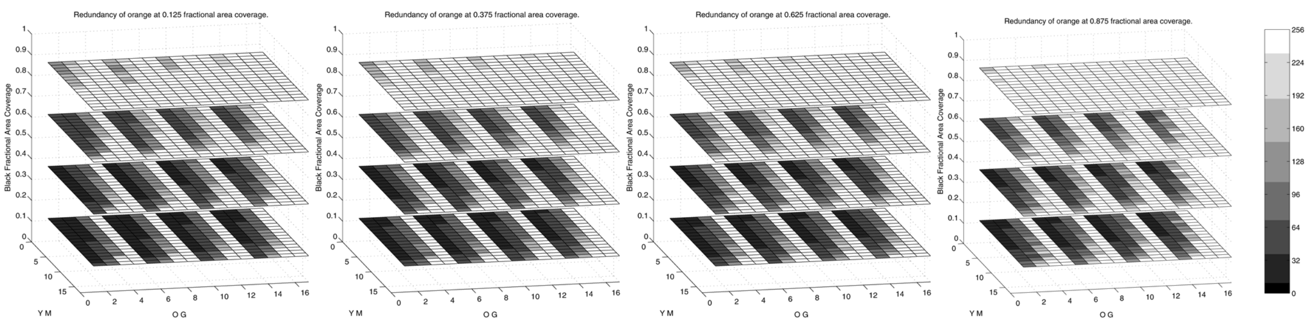


*Figure 19. Orange redundancy density map. Gray level indicates the range of Orange digital counts that can match original midpoints spectra at an RMS difference of 0.02 or less. See Figure 13 for positional interpretation information.*

**Table IV. Summary of Figures 14 - 19**

Figure **F** shows to an RMS tolerance of 0.02, that ink combinations exist allowing for any level of ink $\mathbf{I}_1$ for matching spectra where midpoint ink(s) $\mathbf{I}_2$ are elevated.

| F | $\mathbf{I}_1$ | $\mathbf{I}_2$ |
|---|---|---|
| 14 | cyan | cyan |
| | | combined black and green |
| | | combined magenta and green |
| 15 | magent a | magenta |
| | | black |
| | | combined orange and cyan |
| 16 | yellow | yellow |
| | | orange |
| | | black |
| 17 | black | combined black and green |
| | | combined magenta, cyan and orange |
| | | combined green, cyan and orange |
| | | combined green, magenta and orange |
| 18 | green | green |
| | | cyan |
| 19 | orange | orange |
| | | black |

## Discussion and Conclusions

Figures 14 - 19 and Table IV illustrate the fact that throughout ink space there are many situations in which the same spectrum can be approximately matched by a multitude of ink combinations. Table IV summarizes observations of systematic relationships in these figures. The table shows that when ink levels are high for an original $\mathbf{I}_2$ ink combination, the inks in the $\mathbf{I}_1$ column can be swapped in or out to any desired level and ink combinations exist to match reflectance within 0.02 RMS difference. $\mathbf{I}_2$ ink combinations are from the midpoint set and thus always have at least 0.125 fractional area coverage from every one of the six inks.

Analyses of Figure 9 and Table III showed that within an RMS tolerance of 0.02, spectral reflectances of the pure black ink ramp could be matched by combinations of the other five inks. Further, Figure 12 provides similar evidence for reflectances of black printed in combination with other inks. Thus, if spectral RMS were chosen as a spectral matching metric and the tolerance set at 0.02 RMS difference, then black ink in our systems no longer provides an additional degree of spectral matching freedom. At a tolerance level of 0.01 RMS spectral difference, that conclusion does not hold.

One must be careful to choose spectral metrics and tolerances appropriate to their application. Spectral RMS difference may not be adequate to evaluate particular systems. A tolerance of 0.02 RMS difference may, likewise, be too high for particular applications. The same approaches described in this paper, may be used for any chosen metric and any set of tolerances.

Even when analysis shows an ink to be spectrally unnecessary for spectral matching, there might still be other sound reasons to use the ink. For example, some approaches to spectral reproduction break a 6-ink system into logical sets of multiple 4-ink systems[4,5] – in which it may continue to be advantageous to maintain certain inks in the system such as black. Also. there might be reasons similar to the common ones cited for justifying adding black to a CMY colorimetric reproduction system such as cost of colorant and stability of the reproduction process.

The colorimetric analogy, though, breaks down for the spectral situation. For a CMY printer, black will also increase the colorimetric gamut in the darks. It should be emphasized that, within an RMS 0.02 tolerance, the black inks in the evaluated systems add **no increase** to the spectral gamut. They are, instead, completely redundant

There is much spectral redundancy within the evaluated CMYKGO 6-ink ink-jet systems. Investigations were able to map out the density of it throughout the ink space. The relationships found in this system can be utilized to improve spectral color management. Future systems can be designed to avoid or enhance spectral redundancy, based on application needs.

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## Biography

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